A DISPERSIVE GRADIENT ELASTICITY MODEL DERIVED FROM A DISCRETE MICROSTRUCTURE INCLUDING HIGHER-ORDER STIFFNESS AND HIGHER-ORDER MASS

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Standard continuum models are not suitable when phenomena are modeled that are driven by processes on a smaller material scale. For instance, wave dispersion in granular media cannot be modeled by a standard continuum. Also the simulation of strain-softening phenomena cannot be performed with standard continua due to a loss of well-posedness.

As an alternative, it has been proposed to include higher order spatial gradients of one or more state variables in the continuum description of the material. As such, explicitly or implicitly defined length scales enter the formulation in the higher-order constitutive coefficients. These intrinsic length scales are assumed to be a representation of the underlying microstructure. However, not in all types of higher-order gradient models a rigorous relation between the microstructural properties and the intrinsic length scales can be established. In fact, the higher-order gradients normally enter the continuum description in two ways, namely either via postulation or via derivation from a discrete microstructure. Obviously, the latter methodology enables a straightforward identification of the intrinsic length scale(s) in terms of microstructural properties, and is thus to be preferred.

In this contribution we focus on gradient models that are derived from a discrete microstructure. A straightforward continualization of the microstructure leads to anomalies in the description of wave dispersion, which become manifest in two ways: either imaginary phase velocities appear which lead to instabilities, or infinite group velocities appear which lead to an unrealistic propagation of the energy. In this context, it is noted that most phenomenological gradient elasticity models from the literature also exhibit the drawback of unrealistic behavior in dynamics.

To overcome the above-mentioned drawbacks, a new continualization method is proposed to derive a higher-order continuum model from a discrete microstructure. As a result of this method, each higher-order stiffness term is accompanied by a higher-order inertia term. Thus, the obtained models are denoted as dynamically consistent. It is shown that the dynamically consistent gradient models are capable of smoothing singularities (which is crucial in the simulation of softening phenomena) as well as describing wave dispersion with finite propagation velocity of energy and waves. Thus, physically realistic behaviour is obtained in statics as well as in dynamics. Energy considerations reveal that unconditional stability is guaranteed.

Whereas the higher-order stiffness terms lead to the inclusion of internal length scales, in a similar manner internal time scales are associated with the higher-order inertia terms. The effect of the higher-order time scales on the stability of explicit time integration will be investigated. Furthermore, attention will be focused on the spatial discretization. The appearance of higher-order spatial gradients in the balance equations requires a higher-order continuity of the shape functions. Several methods to either achieve this or to circumvent this will be discussed.